Conclusion

The differential equation for normal stress in the adhesive of a lap joint as proposed by Ojalvo and Eidinoff is an improvement over that proposed by Goland and Reissner in that the differential equation of normal stress by Goland and Reissner is based upon an incomplete shear stress-displacement relationship, while that of Ojalvo and Eidinoff is based on the complete relationship. However, the differential equation for shear stress in the adhesive, using a complete shear stress-displacement relationship and assumptions common to the theories of both Goland and Reissner and Ojalvo and Eidinoff is not unique in that that it contains an arbitrary constant. Examination of a wide range of values of that constant on a sample problem indicates that shear stress values vary significantly with the choice of the constant. Different choices of the constant give either the differential equation of Goland and Reissner or of Ojalvo and Eidinoff.

References

[1] Goland, M. and Reissner, E., "The Stresses in Cemented Joints," *Journal of Applied Mechanics,* Vol. 11, March 1944, pp. A17-A27.

[2] Ojalvo, I.U. and Eidinoff, H.L., "Bond Thickness Effects Upon Stresses in Single-Lap Adhesive Joints," *AIAA Journal* Vol. 16, March 1978, pp. 204-211.

Reply by Authors to W.C. Carpenter

I. U. Ojalvo* and H. L. Eidinoff†
Grumman Aerospace Corp., Bethpage, N.Y.

THE purpose of this Reply is to refute the main point of Carpenter's Comment, i.e., that there are arbitrary coefficients in our lap joint adhesive theory as presented in Ref 1. It is our contention that there is a basic error in Carpenter's theory associated with the satisfaction of transverse deflection compatibility at the bond/adherend interface. The following comments contain a development of this position.

The first two arbitrary constants introduced by Carpenter in Eq. (14) of Table 1 contain no explanation other than a footnote which states "α_1 and α_2 are constants which depend upon the shear stress-displacement equation for the adhesive." We shall demonstrate here that α_1 and α_2 are not at all arbitrary, but must be fixed values if the adhesive displacements u_a and w_a are to be compatible with the adherend displacements $\bar{u}_i$ and w_i at their contacting surfaces ($z = \pm h/2$, $|x| < c$). These symbols are defined in Figs. 1 and 2. It should be noted that u_i (undefined by Carpenter) is called $\bar{u}_i$ here and in Ref. 1.

The theory of Ref. 1 was based upon a number of basic assumptions. The assumptions regarding the adhesive which are required for our present Reply are: 1) the adhesive strain-displacement equation is linear, 2) the adhesive material is linearly elastic, and 3) the deflections vary linearly through the bond thickness. Assumptions 1 and 2 were implicitly stated as Eqs. (11) and (12) of Ref. 1, while the third assumption was explicitly stated in that work.

For compatability of adhesive and adherend displacements at $z = \pm h/2$, assumption 3 requires that

$$u_a = (\bar{u}_1 + \bar{u}_2)/2 + (z/h)(\bar{u}_1 - \bar{u}_2) \qquad (1)$$

and

$$w_a = (w_1 + w_2)/2 + (z/h)(w_1 - w_2) \qquad (2)$$

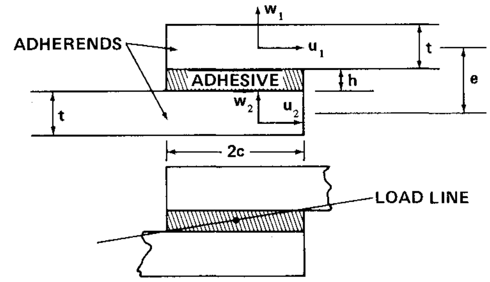
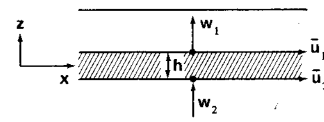


Fig. 1 Unloaded joint—eccentricity, $t+h$.



$$u_a = \frac{\bar{u}_1 + \bar{u}_2}{2} + \frac{z}{h}(\bar{u}_1 - \bar{u}_2)$$

$$w_a = \frac{w_1 + w_2}{2} + \frac{z}{h}(w_1 - w_2)$$

Fig. 2 Adhesive displacements—linear variation through the thickness.

which are identical to Eqs. (10a and b) of Ref. 1.

Combining assumption 1 with Eqs. (1) and (2) yields

$$\gamma_a = \frac{\partial u_a}{\partial z} + \frac{\partial w_a}{\partial x} = \frac{\bar{u}_1 - \bar{u}_2}{h} + \frac{w_1' + w_2'}{z} + \frac{z}{h}(w_1' - w_2') \qquad (3)$$

Combination of Eq. (3) with assumption 2 yields

$$\tau_a = G\gamma_a = G\left(\frac{\bar{u}_1 - \bar{u}_2}{h} + \frac{w_1' + w_2'}{z} + \frac{z}{h}(w_1' - w_2')\right) \qquad (4)$$

which differs from Carpenter's Eq. (14) in that α_1 and α_2 do not appear. It is seen that the only way in which his Eq. (14) will agree with Eq. (4) above is if α_1 and α_2 are both unity, in which case they are fixed and not arbitrary.

The two remaining arbitrary constants in Carpenter's "Comment," α_3 and α_4, appear in a somewhat mysterious fashion in his Eqs. (24) and (26). These terms arbitrarily multiply terms which are identities and thus zero [see his Eqs. (23) and (25)] and are appended to Eq. (22) to become his Eq. (27). He then proceeds to arbitrarily introduce Eq. (28) with no explanation. Presumably, he has done this to reconcile the fact that our technical theory contains an inconsistency. It is our contention that we have admitted this inconsistency in Ref. 1 in our discussion following our Eq. (27c). Such inconsistencies arise from the fact that a displacement field was assumed (for the adhesive) which did not satisfy the adhesive equilibrium equations. The justification for our assumption was that 1) the transverse shear carried by the adhesive was negligible compared to that carried by the adherends and 2) the displacement field chosen was the simplest consistent one possible. We feel Carpenter should have offered some rationale for selecting α_3 and α_4 such as a strain energy (U) formulation in which perhaps U is minimized relative to variations in α_3 and α_4.

In conclusion, it is our position that Carpenter is in error in stating that α_1 and α_2 are arbitrary since deflection compatibility is violated if these coefficients are not equal to unity, and that he has introduced arbitrary constants, α_3 and α_4, which should be justified on some physical grounds.

References

[1] Ojalvo, I.U. and Eidinoff, H.L., "Bond Thickness Effects upon Stresses In Single-Lap Adhesive Joints," *AIAA Journal,* Vol. 16, March 1978, pp. 204-211.

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*Structural Mechanics Group Leader.
†Structural Analysis Engineer.